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Cyclical adjustment of capital requirements: A simple framework

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ABSTRACT

We present a model of an economy with heterogeneous banks that may be funded with uninsured deposits and equity capital. Capital serves to ameliorate a moral hazard problem in the choice of risk. There is a fixed aggregate supply of bank capital, so the cost of capital is endogenous. A regulator sets risk-sensitive capital requirements in order to maximize a social welfare function that incorporates a social cost of bank failure. We consider the effect of a negative shock to the supply of bank capital and show that optimal capital requirements should be lowered. Failure to do so would keep banks safer but produce a large reduction in aggregate investment. The result provides a rationale for the cyclical adjustment of risk-sensitive capital requirements.

1. Introduction

Discussions on the potential business cycle amplification effects of Basel II started long before its approval in 2004 by the Basel Committee on Banking Supervision (BCBS, 2004). The argument whereby these effects may occur is well-known. In recessions, losses erode banks' capital, while risk-sensitive capital requirements such as those in Basel II become higher. If banks cannot quickly raise sufficient new capital, they will be forced to reduce their lending, thereby contributing to the worsening of the downturn. However, a reduction in capital requirements makes banks riskier, so there is a trade-off.

The purpose of this paper is to construct a simple model of optimal capital regulation that illustrates this trade-off. The model has a continuum of banks that differ in an observable characteristic (their "risk type") that is related to their incentives to take risk. Banks may fund their investments

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with uninsured deposits and equity capital. There is a moral hazard problem in the choice of risk that implies inefficient risk-shifting under debt finance, which capital serves to ameliorate. A regulator sets risk-sensitive capital requirements in order to maximize a social welfare function that incorporates a social cost of bank failure. This yields a capital charge curve that is increasing in the banks' risk type. We consider a short-run situation (or one with severe capital market frictions) in which bank capital is exogenously fixed, and study the effects of a negative shock to the aggregate supply of bank capital.[1] We show that the optimal response to the shock is to lower capital requirements. Failure to do so would keep banks safer but produce a large reduction in aggregate investment. The result provides a rationale for the cyclical adjustment of risk-sensitive capital requirements.

The paper is closely related to Kashyap and Stein (2004). They present a framework (which is developed in the longer working paper version of their article) in which there is a regulator that cares about bank lending as well as the social cost of bank failure. They conclude that "instead of there being a single once-and-for-all curve that maps risk measures into capital charges, optimality requires a family of point-in-time curves, with each curve corresponding to (...) different macroeconomic conditions." In their model there is a representative bank that maximizes the expected return of a portfolio of different types of risky loans. There is also a regulator that maximizes the expected return of the bank's portfolio minus a reduced-form term that captures the social cost of bank failure. The regulator chooses capital requirements for each type of loan in order to maximize its objective function subject to a capital availability constraint. The shadow value of bank capital is the Lagrange multiplier associated to this constraint. They conclude that when bank capital is scarce, its shadow value will be high, and the regulator should lower capital requirements.

Although their intuition is the same as ours, the models are very different. Kashyap and Stein do not consider the effect of limited liability, ignoring that the convexity of the bank's objective function implies that it would want to specialize in only one type of loans (see Repullo and Suarez, 2004). They also take as exogenous the risk-adjusted discount rate for each type of loan, a variable that should in principle depend on the (endogenous) capital requirement for each type of loan. Finally, they model in a reduced-form manner the effect of capital on the probability of bank failure.

In contrast, our approach does not suffer from these shortcomings. Building on Repullo (2005), in our model a continuum of banks with different risk types have an investment opportunity of size one that may be funded by risk-neutral depositors and outside equity investors. There is an infinitely elastic supply of uninsured deposits at an expected return that is normalized to zero and a fixed aggregate supply of bank capital, so the cost of capital is endogenously determined in equilibrium. After raising the required funds, each bank chooses a risk parameter that, together with its type, determines its probability of failure. The bank's choice of risk is not observed by depositors, so there is a (risk-shifting) moral hazard problem.

We first characterize the equilibrium of the model in the absence of regulation. Interestingly, banks will in general want to have capital in order to ameliorate the moral hazard problem. The trade-off is that capital helps on the moral hazard front, but it is in general more expensive than deposits. In fact, when the cost of capital equals the return required by depositors there is no trade-off, and banks would only be funded with equity.

We then introduce a risk-neutral regulator that faces the same informational constraints as the market, in particular the inability to observe the banks' choice of risk. For this reason, the regulator resorts to using capital requirements to indirectly influence banks' risk-taking. Unlike in the Basel II regulation, which is based on targeting an exogenous probability of failure for all banks, here the regulator maximizes society's welfare subject to the capital availability constraint. The social welfare function incorporates a term that captures the negative externalities associated with bank failures. Of course, if bank failures entailed no social cost, the market equilibrium would be efficient, and bank capital regulation would not be justified. In contrast, when there is a social cost of bank failure, the regulator requires banks to have more capital than they would choose in the absence of regulation. But there is a trade-off: although banks will be safer, aggregate investment will be lower. We show

[1] This is the same approach as in Holmström and Tirole (1997).

that the optimal regulation may be implemented as a risk-based schedule of minimum capital requirements, with banks of riskier types facing higher capital requirements.

Finally, we consider the effect of a negative shock to the aggregate supply of bank capital, which could be interpreted as the result of a downturn of the economy that produces losses that erode banks' capital. Obviously, our modelling approach implicitly assumes the existence of capital market imperfections that make it impossible for banks to raise new capital. We show that the shock increases the shadow value of bank capital and consequently reduces optimal capital requirements. We also show that if capital requirements are kept unchanged, the reduction in the supply of bank capital will be accommodated by a significant reduction in bank lending and aggregate investment. However, the corresponding reduction in social welfare is mitigated by the fact that the operating banks will be safer than in the optimal regulation.

The literature on the procyclical effects of risk-sensitive bank capital regulation has grown in recent years. The closest paper is Repullo and Suarez (2013). In contrast with our static setup, they consider a dynamic model of relationship lending in which banks are unable to access the equity markets every period and the business cycle is modeled as a two-state Markov process that determines the loans' probabilities of default. They compare the performance of several capital regulation regimes, including one that maximizes social welfare. Their analysis is complicated by the fact that to protect their future lending capacity, banks will in general choose to have capital in excess of the minimum required by regulation. They show that the risk-based requirements of Basel II are more procyclical than the flat requirements of the earlier Basel I regulation, but make banks safer. They also show that Basel II dominates Basel I in terms of social welfare except for low values of the social cost of bank failure. In contrast with our static model, in their dynamic model shocks to bank capital come from defaults of past loans. However, they do not have a cross-sectional distribution of bank risks, since all the loans granted in any period have the same probability of default.

Other related literature includes the early contributions of Daníelsson et al. (2001) and Gordy and Howells (2006), and the more recent of Brunnermeier et al. (2009), Hanson et al. (2011), and Shleifer and Vishny (2010), which note the potential importance of the procyclical effects of risk-sensitive capital requirements and elaborate on the pros and cons of the various policy options for their correction.

The procyclicality problem received considerable attention in statements of the G-20 following the failure of Lehman Brothers.[2] The 2010 agreement of the Basel Committee (BCBS, 2010a), known as Basel III, refers to the following four key objectives: dampen any excess cyclically of the minimum capital requirement, promote more forward looking provisions, conserve capital to build buffers that can be used in stress, and achieve the broader macroprudential goal of protecting the banking sector from periods of excess credit growth. However, there is essentially nothing in Basel III on the first two objectives.[3] The third objective gave rise to the capital conservation buffer, and the fourth to the countercyclical capital buffer. While the capital conservation buffer is a reasonable proposal in the spirit of prompt corrective action provisions of the 1992 Federal Deposit Insurance Corporation Improvement Act (FDCIA), Repullo and Saurina (2012) argue that the proposed capital conservation buffer (see BCBS, 2010b) might actually exacerbate the procyclical effects of the regulation, because the variable on which it is based (the credit-to-GDP gap) tends to be negatively correlated with GDP growth.

The rest of the paper is organized as follows. Section 2 presents the model and characterizes the equilibrium in the absence of regulation. Section 3 introduces a social cost of bank failure and characterizes the optimal bank capital regulation. Section 4 provides a numerical illustration of the previous results. Section 5 discusses the effects of a negative shock to the aggregate supply of bank capital under optimally adjusted and fixed capital requirements. Section 6 concludes. Appendix A shows that the results are robust to the introduction of an elastic aggregate supply of bank capital, and Appendix B contains the proofs of the analytical results.

[2] For example, in the November 2008 Washington Summit the G-20 instructed the International Monetary Fund (IMF), the Financial Stability Forum (FSF), and the Basel Committee "to develop recommendations to mitigate procyclicality, including the review of how valuation and leverage, bank capital, executive compensation, and provisioning practices may exacerbate cyclical trends."

[3] To mitigate the excess cyclicality of the minimum capital requirement, Repullo et al. (2010) propose to use a business cycle multiplier that would be an increasing function of GDP growth.

2. The model

Consider an economy with two dates ($t = 0, 1$), a continuum of *risk-neutral banks* described by their (observable) type $\theta \in [0, 1]$, and a large set of *risk-neutral investors* that can fund the banks with uninsured deposits and outside equity capital. The distribution of potential bank types θ is assumed to be uniform in the interval [0, 1].

At $t = 0$ a bank of type θ can invest one unit of funds in a risky asset that yields a *stochastic payoff* at $t = 1$ given by

$$R = \begin{cases} \max\{a(2\theta - p), 0\}, & \text{with probability } p, \\ 0, & \text{with probability } 1 - p, \end{cases} \tag{1}$$

where $a > 1$ is a parameter that characterizes the profitability of the banks' investments, and $p \in [0, 1]$ is a parameter privately chosen by the bank at $t = 0$, which is the source of the (risk-shifting) moral hazard problem.[4] Notice that higher risk (lower p) is associated with a higher success payoff.[5]

The functional form in (1) implies

$$\theta = \arg\max_p p[a(2\theta - p)].$$

This means that in the absence of moral hazard, a bank of type θ would choose $p = \theta$, which is the (first-best) probability of success that maximizes the bank's expected payoff. For this reason, we will refer to banks with high (low) θ's as safer (riskier) banks.

Banks may fund their investment by raising funds from uninsured depositors, that require an expected return that is normalized to 0, and from outside equity investors, that require an expected (excess) return $\delta \geqslant 0$.[6] We assume that there is a *fixed aggregate supply of bank capital* $\overline{K}$, so the cost of capital δ will be endogenously determined.

In the absence of regulation, banks choose at $t = 0$ the amount of capital $k \in [0, 1]$ and deposits $1 - k$, as well the (gross) interest rate b offered to the depositors and the ownership share $\alpha \in [0, 1]$ offered to the outside shareholders, so an ownership share $1 - \alpha$ is retained by the inside shareholders who manage the bank.

For a given cost of capital δ, the *optimal financing contract* for a bank of type θ is a solution ($k(\theta, \delta)$, $b(\theta, \delta)$, $\alpha(\theta, \delta)$, $p(\theta, \delta)$) to the following problem

$$\max_{(k,b,\alpha,p)} (1 - \alpha)p[a(2\theta - p) - b(1 - k)] \tag{2}$$

subject to the *incentive compatibility constraint*

$$p(\theta, \delta) = \arg\max_p p[a(2\theta - p) - b(\theta, \delta)(1 - k(\theta, \delta))], \tag{3}$$

the *depositors' participation constraint*

$$p(\theta, \delta)b(\theta, \delta) = 1, \tag{4}$$

and the *outside shareholders' participation constraint*

$$\alpha(\theta, \delta)p(\theta, \delta)[a(2\theta - p(\theta, \delta)) - b(\theta, \delta)(1 - k(\theta, \delta))] = (1 + \delta)k(\theta, \delta). \tag{5}$$

The objective function in (2) is the expected payoff of the inside shareholders, which equals their ownership share $1 - \alpha$ multiplied by the probability of success p and by the difference between the success return $a(2\theta - p)$ and the promised debt repayment $b(1 - k)$. The incentive compatibility constraint (3) characterizes the bank's choice of p given the repayment $b(\theta, \delta)(1 - k(\theta, \delta))$. The depositors' and the outside shareholders' participation constraints (4) and (5) ensure that they get the required expected return on their investments in the bank.

[4] The max{·, 0} operator ensures that the success payoff is always nonnegative.

[5] This setup is borrowed from Allen and Gale (2000) and is essentially the moral hazard model in Stiglitz and Weiss (1981).

[6] Notice that the maximum expected payoff of the investment of a bank of type θ is $\theta\,[a(2\theta - \theta)] = a\theta^2$. The assumption $a > 1$ implies that in the absence of moral hazard banks with types $\theta \geqslant a^{-1/2}$ would be able to fund their investments with deposits.

The following result characterizes the banks' capital and risk decisions for a given cost of capital δ.

Proposition 1. *The capital and risk decisions of a bank of type θ when the cost of capital is $\delta \geqslant 0$ are*

$$k(\theta,\delta) = 1 - \frac{a\theta^2}{2}\left[1 - \frac{1}{(1+2\delta)^2}\right], \tag{6}$$

$$p(\theta,\delta) = \frac{\theta}{2}\left[1 + \frac{1}{1+2\delta}\right]. \tag{7}$$

Only banks with types $\theta \geqslant \theta(\delta)$, where

$$\theta(\delta) = \sqrt{\frac{1+2\delta}{a(1+\delta)}}, \tag{8}$$

will operate.

The level of capital $k(\theta, \delta)$ chosen by the banks is decreasing in their type θ (so safer banks have less capital) and in the cost of bank capital δ (so banks economize on capital when it becomes more expensive). In the limit case $\delta = 0$, where the cost of bank capital equals the expected return required by depositors, we have $k(\theta, 0) = 1$, that is all banks will be 100% equity financed. The intuition for this result is straightforward. Bank capital helps to ameliorate the risk-shifting problem but it is in general more expensive than deposits, except in the limit case $\delta = 0$ where there is no trade-off, and hence banks choose to be fully funded with equity.

The probability of success $p(\theta, \delta)$ chosen by the banks is increasing in their type θ (so banks with high θ's are indeed safer) and is decreasing in the cost of bank capital δ (so when banks economize on capital they become riskier). In the limit case $\delta = 0$, where banks are 100% equity financed, we have $p(\theta, 0) = \theta$, which is the first-best probability of success.

The depositors' participation constraint (4) implies

$$b(\theta,\delta) = \frac{1}{p(\theta,\delta)},$$

which means that the effects of θ and δ on the deposit rate $b(\theta, \delta)$ have the opposite sign of their effects on the probability of success $p(\theta, \delta)$. In other words, safer banks either by nature (high θ) or by choice (low δ) pay lower deposit rates.

Finally, the type $\theta(\delta)$ of the marginal bank (whose inside shareholders are indifferent between operating and not operating it) is increasing in the cost of bank capital δ. Hence an increase in δ reduces the set of banks that operate in the economy (of types $\theta \in [\theta(\delta), 1]$) and also reduces the demand for capital of the operating banks. This means that the aggregate demand for bank capital

$$K(\delta) = \int_{\theta(\delta)}^{1} k(\theta,\delta)d\theta \tag{9}$$

will be decreasing in the cost of capital δ.

The *equilibrium cost of bank capital* $\hat{\delta}$ is found by equating the aggregate demand for bank capital $K(\delta)$ to the fixed supply $\overline{K}$, that is by solving the equation

$$K(\hat{\delta}) = \overline{K}. \tag{10}$$

We are going to assume that the aggregate supply of bank capital $\overline{K}$ is such that $\hat{\delta} > 0$. By Proposition 1 this requires

$$\int_{\theta(0)}^{1} k(\theta,0)d\theta = \int_{a^{-1/2}}^{1} d\theta = 1 - a^{-1/2} > \overline{K},$$

which may be rewritten as

$$a(1-\overline{K})^2 > 1. \tag{11}$$

Since each operating bank invests a unit of funds, *aggregate investment* in this economy is equal to the mass of banks that operate in equilibrium, that is

$$\widehat{I} = 1 - \hat{\theta},$$

where $\hat{\theta} = \theta(\hat{\delta})$. Given that $K(\delta)$ is decreasing and $\theta(\delta)$ is increasing in δ, it follows that a contraction in the supply of bank capital $\overline{K}$ will increase the equilibrium cost of bank capital $\hat{\delta}$ and reduce aggregate investment $\widehat{I}$ in the economy.

An interesting feature of this model, which contrasts with many models in the banking literature, is that banks will voluntarily choose to have a positive level of capital $k(\theta, \delta) > 0$. There are two reasons for this result. First, having capital k reduces the required amount of deposits $1 - k$, which ameliorates the risk-shifting problem generated by debt finance. Second, this effect reduces the interest rate b of uninsured deposits, and hence the face value $b(1 - k)$ of the debt to be repaid at $t = 1$,which further ameliorates the risk-shifting problem.

3. Optimal bank capital regulation

To motivate bank capital regulation we are going to consider that bank failures entail a social cost. A convenient parameterization is to assume that for a bank of type θ this cost is equal to $ca\theta$, that is a proportion $c > 0$ of the success payoff of the bank's investment under the first-best probability of success $p = \theta$, which is $a(2\theta - p) = a\theta$. Since this cost is not internalized by the banks, their choice of capital and risk will be socially inefficient.

To deal with this externality, we introduce a *risk-neutral regulator* whose objective function is to maximize social welfare. The regulator faces the same informational constraints as the market, in particular the inability to directly control banks' risk-taking, so it resorts to using capital requirements to indirectly influence banks' choice of risk. To get interior solutions to the optimal capital requirements, we assume that parameter c satisfies[7]

$$c < \frac{2[a(1 - \overline{K})^2 - 1]}{1 + 2a\overline{K}(1 - \overline{K})}. \tag{12}$$

In our risk-neutral economy, *social welfare* is measured by the sum of the expected payoffs of depositors and bank (inside and outside) shareholders, minus the expected social cost associated with bank failures. But since depositors receive the required return on their contribution to banks' financing, we can ignore their payoff in the welfare calculations.

The *optimal capital requirements* are obtained as a solution $(k^*(\theta), b^*(\theta), p^*(\theta), \theta^*)$ to the following problem

$$\max_{(k(\theta),b(\theta),p(\theta),\theta^*)} \int_{\theta^*}^{1} [p[a(2\theta - p) - b(1 - k)] - (1 - p)ca\theta] d\theta \tag{13}$$

subject to the *incentive compatibility constraint*

$$p^*(\theta) = \arg\max_p p[a(2\theta - p) - b^*(\theta)(1 - k^*(\theta))], \quad \text{for all } \theta, \tag{14}$$

the *depositors' participation constraint*

$$p^*(\theta)b^*(\theta) = 1, \quad \text{for all } \theta, \tag{15}$$

and the *capital availability constraint*

$$\int_{\theta^*}^{1} k(\theta) d\theta = \overline{K}. \tag{16}$$

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[7] Note that condition (11) implies that the right-hand side of condition (12) is positive.

The integrand of the regulator's objective function (13) has two components: The first one is the banks' expected profits and the second one, with negative sign, is the expected social cost of bank failure. The integral ranges from θ^* (the type of the riskiest bank that is allowed to operate) to 1 (the type of the safest bank). In choosing the optimal capital requirement $k^*(\theta)$ for each type of bank $\theta \geqslant \theta^*$, the regulator takes into account that the bank will be optimally setting the deposit rate $b^*(\theta)$ to raise the required deposits $1 - k^*(\theta)$. This explains the incentive compatibility constraint (14) and the depositors' participation constraint (15), which are identical to the constraints (3) and (4) in the case of the unregulated bank. The regulator also takes into account the overall availability of bank capital in constraint (16).

Since the first-order condition that characterizes the solution to the incentive compatibility constraint (14) is

$$a(2\theta - p^*(\theta)) - b^*(\theta)(1 - k^*(\theta)) = ap^*(\theta), \tag{17}$$

the objective function may be written as

$$\int_{\theta^*}^{1} [ap^2 - (1 - p)ca\theta] d\theta \tag{18}$$

The following result characterizes the optimal capital requirements.

Proposition 2. *If c satisfies condition (12), the optimal capital requirements and corresponding risk decisions for a bank of type θ are*

$$k^*(\theta) = 1 - \frac{a\theta^2}{2}\left[1 - \left(\frac{1+c}{2\lambda - 1}\right)^2\right], \tag{19}$$

$$p^*(\theta) = \frac{\theta}{2}\left[1 + \frac{1+c}{2\lambda - 1}\right], \tag{20}$$

where λ is the Lagrange multiplier associated with the capital availability constraint (16). The values of λ and the type θ^* of the marginal bank are obtained as the unique solution to the system formed by the capital availability constraint (16) and the condition

$$a(p^*(\theta^*))^2 - (1 - p^*(\theta^*))ca\theta^* - \lambda k^*(\theta^*) = 0 \tag{21}$$

that the contribution of the marginal bank to social welfare be zero, and satisfy

$$\frac{1+c}{2\lambda - 1} < 1. \tag{22}$$

The Lagrange multiplier λ is the *shadow value of bank capital*, that is the increase in social welfare resulting from a marginal increase in the aggregate supply of bank capital. As in Kashyap and Stein (2004), the optimal capital requirements $k^*(\theta)$ are decreasing in λ. Proposition 2 shows that the Lagrange multiplier λ and the type θ^* of the marginal bank are obtained by solving a system of two equations: The capital availability constraint (16) and the condition (21) that the contribution of the marginal bank to social welfare be zero. The first condition implies a downward sloping relationship between λ and θ^*: If bank capital becomes more valuable, then according to (19) the regulator will lower capital requirements so more banks will be allowed to operate and the type of the marginal bank will be lower. The second condition implies an upward sloping relationship between λ and θ^*: If bank capital becomes more valuable, then the marginal bank must be of a higher type. Hence there is (at most) a unique intersection between the two functions that determines λ and θ^*.

The result (22) implies that the optimal capital requirements $k^*(\theta)$ set by the regulator are decreasing in the bank's type θ (so safer banks are required to have less capital). The corresponding probabilities of success $p^*(\theta)$ chosen by the banks are increasing in their type θ (so banks with high θ's are indeed safer).

The proof of Proposition 2 shows that when the parameter c that characterizes the social cost of bank failure reaches the upper bound in (12), the Lagrange multiplier λ satisfies $(1 + c)/(2\lambda - 1) = 1$, in which case (19) and (20) become $k^*(\theta) = 1$ (100% capital requirements) and $p^*(\theta) = \theta$ (the first-best probability of success). The intuition for this result is clear. When the social cost of bank failure is sufficiently large, the primary objective of the regulator becomes to minimize the probability of bank failure, which obtains when banks are solely financed with equity.

Under the optimal regulation there will be a corresponding *equilibrium cost of bank capital* δ^* determined by the condition that the inside shareholders of the marginal bank of type θ^* must be indifferent between operating and not operating it. Assuming that banks do not want to have more capital than the one required by regulation (this will be shown to be the case in Proposition 3 below), the equilibrium condition in the market for bank capital will coincide with the capital availability constraint (16) in the regulator's problem, so the type of the marginal bank will be θ^*. Hence the equilibrium cost of bank capital δ^* under the optimal regulation will be determined by the condition

$$a(p^*(\theta^*))^2 - (1+\delta^*)k^*(\theta^*) = 0, \tag{23}$$

where the first term in this expression is the expected profits of the marginal bank (using the first-order condition (17)), and the second is the required compensation of the outside shareholders.

The following result compares the equilibrium with and without capital regulation.

Proposition 3. *When the social cost of bank failure is zero the equilibrium allocation in the absence of regulation is optimal. When $c > 0$ we have*

$$k^*(\theta) > k(\theta, \hat{\delta}),$$
$$p^*(\theta) > p(\theta, \hat{\delta}),$$
$$I^* = 1 - \theta^* < 1 - \hat{\theta} = \widehat{I}.$$

Moreover, banks do not want to have more capital than $k^*(\theta)$.

There are three separate results in Proposition 3. The first one states that when there are no externalities associated with bank failures, the unregulated market equilibrium is efficient, with banks privately choosing the optimal amount of capital.[8] In this case we have $\lambda = 1 + \hat{\delta}$, so the shadow value of bank capital equals the equilibrium private cost of bank capital.

The second result states that when bank failures entail a social cost, the optimal regulation requires banks to have more capital than they would in the unregulated market equilibrium, so they become safer. But there is a trade-off: With an exogenously given supply of bank capital fewer banks will be operating, and hence aggregate investment will fall.

The third result relates to the equilibrium cost of bank capital δ^* under the optimal regulation: For this value of the cost of capital, banks would not want to have more capital than the level required by the regulator. This implies that the optimal regulation may be implemented as a risk-based schedule of minimum capital requirements.

4. A numerical illustration

To illustrate our previous results, consider a numerical example in which we set the parameter that characterizes the profitability of the banks' investments $a = 5$, and suppose that the aggregate supply of bank capital $\overline{K}$ is such that the equilibrium cost of bank capital in the absence of regulation is $\hat{\delta} = 12.5\%$.[9]

By Proposition 1 the equilibrium capital and risk decisions of a bank of type θ are

$$k(\theta, \hat{\delta}) = 1 - 0.9\theta^2, \tag{24}$$

$$p(\theta, \hat{\delta}) = 0.9\theta. \tag{25}$$

[8] It is worth noting that this result would not obtain if deposits were insured, because then banks would not want to have any capital.

[9] It should be noted that these and the other parameter values below are not intended to provide a calibration of the model, since they are simply chosen to facilitate the graphical representation of the qualitative results of the paper.

Thus the safest bank (of type $\theta = 1$) will choose a level of capital $k(1, \hat{\delta}) = 10\%$ and a probability of success $p(1, \hat{\delta}) = 90\%$. Riskier banks (with $\theta < 1$) will have more capital, but this will be insufficient to compensate the worsening of the moral hazard problem, and they will choose lower probabilities of success. Also by Proposition 1, the type of the marginal bank that is indifferent between operating and not operating will be $\hat{\theta} = \theta(\hat{\delta}) = 4.5^{-1/2} = 0.471$. Finally, the required aggregate supply of bank capital is given by

$$\overline{K} = \int_{\hat{\theta}}^{1} k(\theta, \hat{\delta}) d\theta = 0.260.$$

To compute the optimal capital requirements we set the social cost of bank failure $c = 0.2$. Solving Eqs. (16) and (21) gives a shadow value of bank capital $\lambda = 1.211$ and a marginal bank type $\theta^* = 0.538$. Hence by Proposition 2 the optimal capital requirements and the corresponding risk decisions for a bank of type θ are

$$k^*(\theta) = 1 - 0.718\theta^2, \tag{26}$$

$$p^*(\theta) = 0.922\theta. \tag{27}$$

Thus the safest bank (of type $\theta = 1$) will face a capital requirement $k^*(1) = 28.2\%$ and will choose a probability of success $p^*(1) = 92.2\%$. Note that, as stated in Proposition 3, $k^*(\theta) > k(\theta, \hat{\delta})$ and $p^*(\theta) > p(\theta, \hat{\delta})$, so banks will have more capital and will be safer than in the absence of regulation. However, given that there is a fixed aggregate supply of bank capital, requiring banks to have more capital will necessarily reduce the set of banks that operate. In particular, the type of the marginal bank will increase from $\hat{\theta} = 0.471$ to $\theta^* = 0.538$. Therefore aggregate investment will fall by 12.6% from $\hat{I} = 1 - \hat{\theta} = 0.529$ to $I^* = 1 - \theta^* = 0.462$. Finally, the equilibrium cost of capital will jump from $\hat{\delta} = 12.5\%$ to $\delta^* = 55.3\%$, reflecting the increase in the demand for bank capital generated by the optimal regulation.

To illustrate the result, Fig. 1 plots the functions $k(\theta, \hat{\delta})$ and $k^*(\theta)$ in (24) and (26). To facilitate the comparison with the standard capital charge curves à la Basel II, the variable in the horizontal axis is $1 - \theta$, which is a measure of banks' risk. The two functions have a similar shape, with the gap between

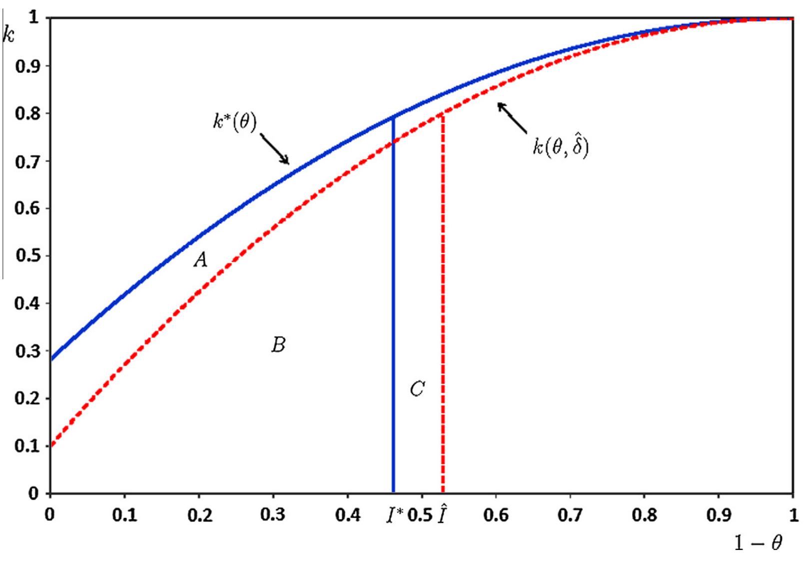

Fig. 1. Equilibrium capital and optimal capital requirements for a fixed supply of bank capital. This figure depicts the equilibrium capital decisions in the absence of regulation and the optimal capital requirements for the different types of banks, with the corresponding levels of aggregate investment in the horizontal axis. The sum of the areas of regions *A* and *B* and the sum of the areas of regions *B* and *C* equals the aggregate supply of bank capital.

$k(\theta, \hat{\delta})$ and $k^*(\theta)$ becoming smaller when θtends to zero. Fig. 1 also shows the critical values $I^* = 1 - \theta^*$ and $\widehat{I} = 1 - \hat{\theta}$ beyond which banks will not be operating, respectively, with and without capital requirements. Under the assumption of a uniform distribution of bank types, the integral below the curve $k(\theta, \hat{\delta})$ between 0 and $\widehat{I}$ equals the aggregate supply of bank capital $\overline{K}$, and similarly the integral below the curve $k^*(\theta)$ between 0 and I^* also equals $\overline{K}$. This means that the area of region A must be equal to the area of region C.

Like in the case of risk-sensitive capital requirements à la Basel II, the optimal capital requirements $k^*(\theta)$ are increasing in the measure of banks' risk, $1 - \theta$. However, our capital requirements are not based on a purely statistical value-at-risk calculation, with an arbitrary confidence level, but follow from the maximization of the appropriate social welfare function.

5. Cyclical adjustment of capital requirements

This section considers the effect of a negative shock to the aggregate supply of bank capital under optimally adjusted and fixed capital requirements.

Specifically, suppose that the supply of bank capital goes down from $\overline{K}_0$ to $\overline{K}_1$. Following the discussion after Proposition 2, we first derive the effect of the shock on the values the Lagrange multiplier λ and the type θ^* of the marginal bank. A reduction in the aggregate supply of bank capital produces an upward shift in the downward sloping relationship between λ and θ^* implied by the capital availability constraint (16). Since the relationship between λ and θ^* implied by the condition (21) on the zero contribution of the marginal bank to social welfare is upward sloping, the effect of the shock will be to increase the value of the Lagrange multiplier λ, reflecting the higher shadow value of bank capital, and the value of the type θ^* of the marginal bank, reflecting the need to shrink the set of banks that will be allowed to operate in order to economize on scarce bank capital.

By Proposition 2, the increase in λ will reduce the optimal capital requirements $k^*(\theta)$ and the probability of success $p^*(\theta)$ of the operating banks. The intuition for these results is clear: The optimal way to accommodate the shock in the aggregate supply of bank capital is to reduce capital requirements in order to avoid the reduction in aggregate investment that otherwise would obtain. The reduction in bank capital in turn explains the increase in the probability of failure of the operating banks. Finally, the increase in the type θ^* of the marginal bank means that aggregate investment will fall, but by less than without the reduction in capital requirements.

We may illustrate these results using our previous numerical example. In particular, suppose that the aggregate supply of bank capital goes down by 25% from $\overline{K}_0 = 0.260$ (the value chosen in Section 4 to get an equilibrium cost of bank capital in the absence of regulation $\hat{\delta} = 12.5\%$) to $\overline{K}_1 = 0.195$. Solving Eqs. (16) and (21) now gives a shadow value of bank capital $\lambda_1 = 1.258$ and a marginal bank type $\theta_1^* = 0.544$. Hence by Proposition 2 the optimal capital requirements and the corresponding risk decisions for a bank of type θ are now given by

$$k_1^*(\theta) = 1 - 0.932\theta^2, \tag{28}$$

$$p_1^*(\theta) = 0.896\theta. \tag{29}$$

Comparing these results with (26) and (27), it follows that the reduction in capital requirements will be very significant, but the effect on bank risk will be relatively small. For example, the capital requirement for the safest bank (of type $\theta = 1$) will be reduced from $k_0^*(1) = 28.2\%$ to $k_1^*(1) = 6.8\%$, while the corresponding probability of success will go down from $p_0^*(1) = 92.2\%$ to $p_1^*(1) = 89.6\%$. The marginal bank will now be of type $\theta_1^* = 0.544$, which means that aggregate investment will only fall by 1.3% from $I_0^* = 1 - \theta_0^* = 0.462$ to $I_1^* = 1 - \theta_1^* = 0.456$. Finally, using (23) we conclude that the equilibrium cost of capital will increase from $\delta_0^* = 55.3\%$ to $\delta_1^* = 64.3\%$, reflecting the negative shock in the aggregate supply of bank capital which is not fully compensated by the reduction in capital requirements.

Fig. 2 plots the optimal capital requirements before and after the shock in the aggregate supply of bank capital, as well as the critical values $I_0^* = 1 - \theta_0^*$ and $I_1^* = 1 - \theta_1^*$ beyond which banks will not be operating, respectively, before and after the shock. As noted above, the adjustment is made by reducing the set of operating banks and by lowering the capital requirements for the banks that remain in

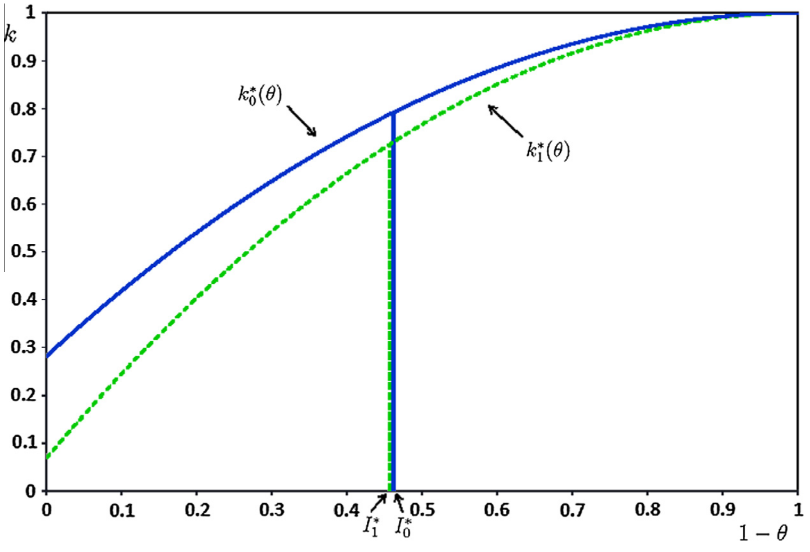

Fig. 2. Optimal capital requirements before and after the shock to the supply of bank capital. This figure depicts the optimal capital requirements for the different types of banks before and after the negative shock to the aggregate supply of bank capital, with the corresponding levels of aggregate investment in the horizontal axis.

operation. In the numerical example, the second element of the adjustment is much more important than the first.

We next consider what happens under a fixed capital requirements regime in which capital requirements are not optimally adjusted following the shock in the aggregate supply of bank capital, but kept fixed at $k_0^*(\theta)$. In this case, the reduction in the supply of bank capital can only be accommodated by a significant reduction in the set of operating banks. Specifically, the type $\tilde{\theta}_1$ of the marginal bank is found by solving the equation

$$\int_{\tilde{\theta}_1}^{1} k_0^*(\theta) d\theta = \overline{K}_1,$$

which gives $\tilde{\theta}_1 = 0.624$. This implies that aggregate investment will fall by 18.6% from $I_0^* = 1 - \theta_0^* = 0.462$ to $\widetilde{I}_1 = 1 - \tilde{\theta}_1 = 0.376$. Finally, to ensure that the inside shareholders of the marginal bank of type $\tilde{\theta}_1$ will be indifferent between operating and not operating it, the equilibrium cost of capital will jump from $\delta_0^* = 55.3\%$ to $\tilde{\delta}_1 = 129.5\%$.

Fig. 3 shows the difference in the adjustment to the shock in the aggregate supply of bank capital when capital requirements are reduced from $k_0^*(\theta)$ to $k_1^*(\theta)$ and when they are kept fixed at $k_0^*(\theta)$. In the first case, aggregate investment goes down to $I_1^* = 1 - \theta_1^* = 0.456$, while in the second it goes down to $\widetilde{I}_1 = 1 - \tilde{\theta}_1 = 0.376$, reflecting the fact that 100% of the reduction in the demand for bank capital is achieved by increasing the cost of capital and consequently reducing the set of operating banks. As before, the integral below the curve $k_1^*(\theta)$ between 0 and I_1^* equals the aggregate supply of bank capital $\overline{K}_1$, and similarly the integral below the curve $k_0^*(\theta)$ between 0 and $\widetilde{I}_1$ also equals $\overline{K}_1$. This means that the area of region *A* must be equal to the area of region *C*. This clearly illustrates the difference in the two adjustment mechanisms: Under the optimal regulation the smaller supply of bank capital is distributed among a larger set of banks, so aggregate investment only falls to I_1^*, while under fixed capital requirements the supply of bank capital is allocated to a smaller set of banks, so aggregate investment falls to $\widetilde{I}_1 < I_1^*$.

Table 1 summarizes the effects of a 25% reduction in the aggregate supply of bank capital on the equilibrium cost of bank capital, aggregate investment, and social welfare in the optimal and the fixed

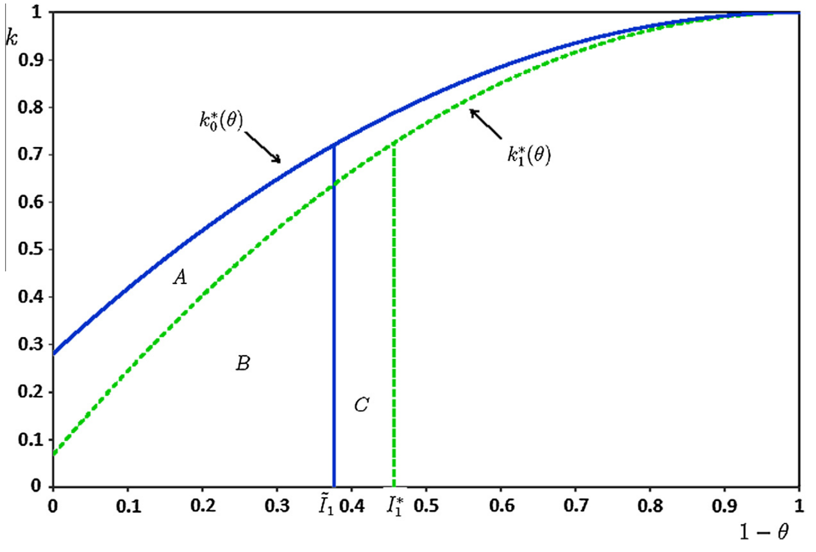

Fig. 3. Adjustment to the shock to the supply of bank capital under fixed and optimal capital requirements. This figure shows the adjustment to the negative shock in the aggregate supply of bank capital when capital requirements are kept fixed and when they are optimally reduced, with the corresponding levels of aggregate investment in the horizontal axis. The sum of the areas of regions *A* and *B* and the sum of the areas of regions *B* and *C* equals the new aggregate supply of bank capital.

Table 1
Effect of a 25% reduction in the supply of bank capital under fixed and optimal capital requirements.

	Initial optimal capital requirements	New optimal capital requirements	Fixed capital requirements
Equilibrium cost of capital (δ), %	55.3	64.3	129.5
Aggregate investment (I)	0.462	0.456	0.376
Social welfare (W)	1.101	1.021	1.001

This table reports the equilibrium cost of bank capital, aggregate investment, and social welfare under optimal capital requirements for the initial aggregate supply of bank capital (column 1) and after a 25 percent reduction in this supply (column 2), as well as the results for the case in which the initial capital requirements are not adjusted (column 3).

capital requirements regimes. Under the optimal regulation the greater part of the adjustment to the new environment is achieved by lowering capital requirements, with only a relatively small increase in the cost of bank capital (which goes from $\delta_0^* = 55.3\%$ to $\delta_1^* = 64.3\%$) and a reduction of only 1.3% in aggregate investment (from $I_0^* = 0.462$ to $I_1^* = 0.456$). Social welfare falls by a greater extent (by 7.3% from $W_0^* = 1.101$ to $W_1^* = 1.021$) because the reduction in capital requirements makes banks riskier, and hence their expected profits go down and the expected social cost of bank failure goes up. In contrast, when capital requirements remain unchanged all the adjustment to the new environment is achieved by increasing the cost of bank capital (which goes from $\delta_0^* = 55.3\%$ to $\tilde{\delta}_1 = 129.5\%$), so there is a very significant reduction in aggregate investment (of 18.6% from $I_0^* = 0.462$ to $\widetilde{I}_1 = 0.376$). Although the operating banks are safer than in the optimal regulation, the reduction in investment leads to a greater fall in social welfare (of 9.1% from $W_0^* = 1.101$ to $\widetilde{W}_1 = 1.001$).

The optimal adjustment of capital requirements yields an increase of 2.0% in social welfare (from $\widetilde{W}_1 = 1.001$ to $W_1^* = 1.021$). This difference may be decomposed as follows:

$$
\begin{aligned}
W_1^* - \widetilde{W}_1 &= \int_{\theta_1^*}^{1} \left[a(p_1^*(\theta))^2 - (1 - p_1^*(\theta))ca\theta\right] d\theta - \int_{\tilde{\theta}_1}^{1} \left[a(p_0^*(\theta))^2 - (1 - p_0^*(\theta))ca\theta\right] d\theta \\
&= \int_{\theta_1^*}^{\tilde{\theta}_1} \left[a(p_1^*(\theta))^2 - (1 - p_1^*(\theta))ca\theta\right] d\theta \\
&\quad - \int_{\tilde{\theta}_1}^{1} a\left[(p_1^*(\theta))^2 - (p_0^*(\theta))^2\right] d\theta - \int_{\tilde{\theta}_1}^{1} \left[p_1^*(\theta) - p_0^*(\theta)\right] ca\theta \, d\theta,
\end{aligned}
$$

where the first term in the last expression is the welfare gain due to the higher investment, the second is the welfare loss due to fact that operating banks choose riskier (and hence less efficient) investments, and the third is the welfare loss due to the higher probability of bank failures. The numerical values of the three terms are

$$
W_1^* - \widetilde{W}_1 = 0.087 - 0.060 - 0.007 = 0.020.
$$

Thus there is an increase in social welfare of 8.7% associated with the higher investment, which is almost compensated by a decrease of 6.0% due to the reduction in the profitability of the operating banks, and a decrease of 0.7% due to the higher social cost of bank failures.[10]

Summing up, our numerical results illustrate the qualitative results of our model, namely that a negative shock to the aggregate supply of bank capital should be partially accommodated by a reduction in capital requirements. Otherwise, banks would be safer but there would be an excessive reduction in the level of economic activity, which would lead to a greater reduction in social welfare.

6. Concluding remarks

This paper presents a simple model of optimal bank capital regulation that provides a rationale for the cyclical adjustment of risk-sensitive capital requirements. Specifically, capital requirements should be lowered in situations where bank capital is scarce such as economic downturns. The trade-off behind the result is explained by Kashyap and Stein (2004) in the following terms: "When banks' lending activities are more severely constrained it is socially desirable to accept a higher probability of bank default (...) It cannot make sense for bank lending to bear the entire brunt of the adjustment, while the expected costs of defaults remain constant."

The results provide a balanced assessment of the costs and benefits of adjusting capital requirements to the state of the business cycle. In particular, from a social welfare perspective it is incorrect either to focus exclusively on the potential credit crunch effects of the regulation, if capital requirements are not lowered in recessions, or to focus exclusively on the greater likelihood of bank failures, if they are. Thus, from a practical point of view, it seems important to integrate a macroprudential with a microprudential perspective. In this regard, the results of the paper are very much in line with those in Repullo and Suarez (2012), who provide "a call for caution against the simple claim that if regulation induces cyclicality it needs to be radically adjusted: the adjustment is not a free lunch."

The results also provide a rationale for the recapitalization of banks with public funds following a negative shock to their capital, as was done in the Capital Purchase Program of the Troubled Assets Relief Program (TARP). If the shadow value of bank capital after the shock is greater than the social cost of public funds, such intervention would be welfare improving.[11]

In contrast with the Basel II regulation, which is based on the value-at-risk criterion that capital must cover losses with a certain confidence level, our model focusses on welfare optimal capital requirements. However, using the results in the proof of Proposition 1, we could easily compute the capital requirements for a confidence level $\gamma \in (0, 1)$,[12] which would be

[10] It should be noted that the difference in welfare terms between adjusting and not adjusting the capital requirements is relatively small. This is explained by the fact that we are taking as reference an initial optimal regulation, so $\lim_{\overline{K}_1 \to \overline{K}_0} \left(W_1^* - \widetilde{W}_1\right) = 0$. I am grateful to Douglas Gale for pointing this out.

[11] I am grateful to Diana Hancock for pointing this out.

[12] Setting $p(\theta, k) = \gamma$ in (33), and solving for k gives $k_\gamma(\theta) = 1 - 2a\gamma(\theta - \gamma)$. The operators max $\{\cdot, 0\}$ and min$\{\cdot, 1\}$ serve to bound the capital requirement between 0 and 1 (and they are in general binding for high and low values of θ, respectively).

$$k_\gamma(\theta) = \min\{\max\{1 - 2a\gamma(\theta - \gamma), 0\}, 1\}.$$

Providing a rationale for a cyclical adjustment of capital requirements would be more complicated in this setup because the safest banks will want to have more capital than the one prescribed by regulation. But the same logic would apply here: Capital requirements designed for good times would be expected to be too high in bad times, so the confidence level γ targeted by the regulator should be adjusted according to the state of the business cycle.

We would like to conclude with two caveats. First, the arrival of a recession may be accompanied by other changes in the model such as reducing the size of the banks' investment opportunities, which was normalized to one, or its profitability, captured by parameter a, or shifting to the left the distribution of bank types. The first effect would reduce the demand for capital, and hence the need for an adjustment of capital requirements, the second would exacerbate the banks' risk-shifting incentives, and hence called for higher rather than lower capital requirements, and the third effect would go in the same direction, since it would reduce the left-hand side of the capital availability constraint (16).

The second caveat is that our setup ignores feedback effects from the level of investment and economic activity to the profitability of the banks' investments. One could introduce these effects by making the profitability parameter a an increasing (and possibly concave) function of the level of aggregate investment I. This would capture demand externalities or technological complementarities similar to those studies in endogenous growth theory. Although the analysis of optimal regulation would be more complicated, it is clear that such effects would strengthen the rationale for the cyclical adjustment of capital requirements.

Acknowledgments

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Appendix A. The model with an elastic supply of bank capital

This Appendix shows that our previous results are robust to the introduction of an upward-sloping aggregate supply of bank capital. Specifically, suppose that supply of capital K is given by

$$K = \overline{K} + \kappa\delta, \tag{30}$$

where δ is the cost of capital, and $\overline{K}$ and κ are positive constants.[13]

To derive the optimal capital requirements we have to modify the regulator's objective function by subtracting the opportunity cost of bank capital, which is given by the triangle area below the supply function

$$\frac{(K - \overline{K})\delta}{2} = \frac{\kappa\delta^2}{2}.$$

The *optimal capital requirements* are obtained as a solution ($k^*(\theta)$, $b^*(\theta)$, $p^*(\theta)$, θ^*, δ^*) to the following problem

$$\max_{(k(\theta), b(\theta), p(\theta), \theta^*, \delta)} \left[\int_{\theta^*}^1 [p[a(2\theta - p) - b(1-k)] - (1-p)ca\theta] d\theta - \frac{\kappa\delta^2}{2} \right]$$

[13] Note that the case with $\kappa = 0$ (which implies $K = \overline{K}$) corresponds to our previous analysis.

subject to the incentive compatibility constraint (14), the depositors' participation constraint (15), the participation constraint (23) of the marginal bank of type θ^*, and the capital availability constraint

$$\int_{\theta^*}^{1} k(\theta)\, d\theta = \overline{K} + \kappa\delta, \tag{31}$$

Following the same steps as in the proof of Proposition 2, we can write the regulator's problem as

$$\max_{(k(\theta),\theta^*,\delta)} \int_{\theta^*}^{1} [a(p(\theta,k))^2 - (1-p(\theta,k))ca\theta - \lambda k]d\theta + \lambda(\overline{K}+\kappa\delta) - \frac{\kappa\delta^2}{2} + \mu[a(p^*(\theta^*))^2 - (1+\delta)k(\theta^*)],$$

where λ denotes the Lagrange multiplier associated with the capital availability constraint (31), μ denotes the Lagrange multiplier associated with the participation constraint (23), and $p\,(\theta, k)$ is given by (33). Differentiating the integrand with respect to k gives a first-order condition whose solution is $k^*(\theta)$ in (19), and substituting this result into (33) and rearranging gives $p^*(\theta)$ in (20). Differentiating objective function with respect to θ^* gives the first-order condition

$$a(p^*(\theta^*))^2 - (1-p^*(\theta^*))ca\theta^* - \lambda k^*(\theta^*) + \mu\left[2ap(\theta^*)\frac{\partial p^*(\theta^*)}{\partial\theta^*} - (1+\delta)\frac{\partial k^*(\theta^*)}{\partial\theta^*}\right] = 0.$$

And differentiating objective function with respect to δ gives the first-order condition

$$\kappa(\lambda-\delta) - \mu k^*(\theta^*) = 0.$$

These two conditions, together with the capital availability constraint (31) and the participation constraint (23), form a system of four equations with four unknowns: the type θ^* of the marginal bank, the cost of capital δ, and the two Lagrange multipliers λ and μ.

To illustrate the results for the model with an elastic supply of bank capital, we set $a = 5$, $c = 0.2$ (the same parameters as before), and $\kappa = 0.1$, and solve for the optimal capital requirements for two different values of the intercept $\overline{K}$ in (30), namely the values $\overline{K}_0 = 0.260$ and $\overline{K}_1 = 0.195$ used in Section 5. The results are given by

$$k_0^*(\theta) = 1 - 0.354\theta^2,$$
$$k_1^*(\theta) = 1 - 0.563\theta^2.$$

Therefore the optimal response to the negative shock in the aggregate supply of bank capital is to lower capital requirements. The marginal bank is of type $\theta_0^* = 0.572$ before the shock and of type $\theta_1^* = 0.574$ after the shock, so aggregate investment will fall by 0.4% from $I_0^* = 1 - \theta_0^* = 0.428$ to $I_1^* = 1 - \theta_1^* = 0.426$. As before, social welfare falls by 6.3% from $W_0^* = 1.155$ to $W_1^* = 1.082$. But if the capital requirements are not optimally adjusted after the shock, but kept fixed at $k_0^*(\theta)$, aggregate investment will fall by 9.5% to $\widetilde{I}_1 = 1 - \tilde{\theta}_1 = 0.387$, and social welfare will fall by 7.1% to $\widetilde{W}_1 = 1.074$. As in the case of the model with an inelastic supply of bank capital, the optimal adjustment of capital requirements yields an increase of only 0.8% in social welfare (from $\widetilde{W}_1 = 1.074$ to $W_1^* = 1.082$), because the welfare gain due to the higher investment is almost compensated by the fact that the operating banks are less profitable and more likely to fail.

Appendix B. Proofs

Proof of Proposition 1. The first-order condition that characterizes the solution to the bank's incentive compatibility constraint (3) is

$$a(2\theta - p) - b(1-k) = ap. \tag{32}$$

Substituting the depositors' participation constraint $pb = 1$ into this expression gives a quadratic equation whose solution is

$$p(\theta,k) = \frac{1}{2}\left(\theta + \sqrt{\theta^2 - \frac{2(1-k)}{a}}\right), \tag{33}$$

where we have chosen the solution with the highest p, which is closest to the first-best $p = \theta$ and hence the one preferred by the bank.

To derive the optimal choice of capital, substitute the outside shareholders' participation constraint (5) and the first-order condition (32) into the bank's objective function (2) to get

$$(1-\alpha)p[a(2\theta-p)-b(1-k)] = p[a(2\theta-p)-b(1-k)]-(1+\delta)k = ap^2-(1+\delta)k. \tag{34}$$

Substituting (33) into this expression and differentiating with respect to k gives the first-order condition

$$\frac{\theta+\sqrt{\theta^2-\frac{2(1-k)}{a}}}{2\sqrt{\theta^2-\frac{2(1-k)}{a}}} = 1+\delta.$$

Solving for k in this condition gives $k(\theta, \delta)$ in (6), and substituting this result into (33) and rearranging gives $p(\theta, \delta)$ in (7).

Finally, substituting $p(\theta, \delta)$ and $k(\theta, \delta)$ into (34) gives

$$a[p(\theta,\delta)]^2-(1+\delta)k(\theta,\delta) = a\theta^2\left(\frac{1+\delta}{1+2\delta}\right)^2-(1+\delta)\left[1-\frac{a\theta^2}{2}\left(1-\frac{1}{(1+2\delta)^2}\right)\right] \geqslant 0,$$

which simplifies to

$$a\theta^2\frac{1+\delta}{1+2\delta} \geqslant 1,$$

Hence the expected payoff of the inside shareholders will be nonnegative for $\theta \geqslant \theta(\delta)$, where $\theta(\delta)$ is given by (8). □

Proof of Proposition 2. Following the same steps as in the proof of Proposition 1, we can solve the first-order condition that characterizes the solution to the bank's incentive compatibility constraint (14) together with the depositors' participation constraint (15) to get a quadratic equation in p whose solution is (33). Then we can write the regulator's problem as

$$\max_{(k(\theta),\theta^*)}\int_{\theta^*}^{1}[a(p(\theta,k))^2-(1-p(\theta,k))ca\theta-\lambda k]d\theta+\lambda\overline{K},$$

where λ denotes the Lagrange multiplier associated with the capital availability constraint (16). Differentiating the integrand with respect to k gives the first-order condition

$$\frac{\theta(1+c)+\sqrt{\theta^2-\frac{2(1-k)}{a}}}{2\sqrt{\theta^2-\frac{2(1-k)}{a}}} = \lambda.$$

Solving for k in this condition gives $k^*(\theta)$ in (19), and substituting this result into (33) and rearranging gives $p^*(\theta)$ in (20).

Differentiating the regulator's objective function with respect to θ^* gives the first-order condition

$$F(\theta^*,\lambda) = a(p^*(\theta^*))^2-(1-p^*(\theta^*))ca\theta^*-\lambda k^*(\theta^*) = 0, \tag{35}$$

which states that the contribution to social welfare of the marginal bank of type θ^* is zero. The values of the Lagrange multiplier λ and the type θ^* of the marginal bank are found by solving (35) together with the capital availability constraint

$$G(\theta^*,\lambda) = \int_{\theta^*}^{1}k^*(\theta)\,d\theta-\overline{K} = 0. \tag{36}$$

To show that these two equations have at most a unique solution it suffices to show that

$$\frac{\partial F(\theta^*,\lambda)}{\partial\theta^*} > 0 \quad\text{and}\quad \frac{\partial F(\theta^*,\lambda)}{\partial\lambda} < 0,$$

so the relationship between λ and θ^* implicit in (35) is increasing, and that

$$\frac{\partial G(\theta^*,\lambda)}{\partial\theta^*} < 0 \quad \text{and} \quad \frac{\partial G(\theta^*,\lambda)}{\partial\lambda} < 0,$$

so the relationship between λ and θ^* implicit in (36) is decreasing. The latter results are immediate from (36) and the expression (19) for $k^*(\theta)$, since

$$\frac{\partial G(\theta^*,\lambda)}{\partial\theta^*} = -k^*(\theta^*) < 0 \quad \text{and} \quad \frac{\partial G(\theta^*,\lambda)}{\partial\lambda} = \int_{\theta^*}^1 \frac{\partial k^*(\theta)}{\partial\lambda}\, d\theta < 0.$$

Next differentiating (35) with respect to θ^* and using the expression (20) for $p^*(\theta)$, Eq. (35), and the expression (19) for $k^*(\theta)$ gives

$$\begin{aligned}\frac{\partial F(\theta^*,\lambda)}{\partial\theta^*} &= 2ap^*(\theta^*)\frac{\partial p^*(\theta^*)}{\partial\theta^*} - \frac{\partial}{\partial\theta^*}[(1-p^*(\theta^*))ca\theta^*] - \lambda\frac{\partial k^*(\theta^*)}{\partial\theta^*}\\ &= \frac{2}{\theta^*}[a(p^*(\theta^*))^2 - (1-p^*(\theta^*))ca\theta^*] + ca - \lambda\frac{\partial k^*(\theta^*)}{\partial\theta^*}\\ &= \frac{2}{\theta^*}\lambda k^*(\theta^*) + ca - \lambda\frac{\partial k^*(\theta^*)}{\partial\theta^*} > 0.\end{aligned}$$

Finally, differentiating (35) with respect to λ and using the expressions (20) for $p^*(\theta)$ and (19) for $k^*(\theta)$ gives

$$\begin{aligned}\frac{\partial F(\theta^*,\lambda)}{\partial\lambda} &= [2ap^*(\theta^*) + ca\theta^*]\frac{\partial p^*(\theta^*)}{\partial\lambda} - \lambda\frac{\partial k^*(\theta^*)}{\partial\lambda} - k^*(\theta^*)\\ &= -\left[2\lambda\frac{a\theta^*(1+c)}{(2\lambda-1)}\right]\frac{\theta^*(1+c)}{(2\lambda-1)^2} + 2\lambda\frac{a(\theta^*)^2(1+c)^2}{(2\lambda-1)^3} - k^*(\theta^*)\\ &= -k^*(\theta^*) < 0.\end{aligned}$$

The upper bound in (12) for c is derived as follows. Suppose that the Lagrange multiplier λ satisfies $(1+c)/(2\lambda-1) = 1$, in which case (19) and (20) become $k^*(\theta) = 1$ and $p^*(\theta) = \theta$. For any constant k (which we are going to set at $k = 1$) the capital availability constraint (16) becomes

$$\int_{\theta^*}^1 k\, d\theta = k(1-\theta^*) = \overline{K},$$

which implies

$$\theta^* = 1 - \frac{\overline{K}}{k}.$$

Differentiating with respect to k the regulator's objective function evaluated at $k = 1$ gives

$$\begin{aligned}\frac{d}{dk}\int_{\theta^*}^1 [a[p(\theta,k)]^2 - [1-p(\theta,k)]ca\theta]d\theta &= \int_{\theta^*}^1 \left(1+\frac{c}{2}\right)d\theta - \left[a(\theta^*)^2 - (1-\theta^*)ca\theta^*\right]\frac{d\theta^*}{dk}\\ &= \left(1+\frac{c}{2}\right)\overline{K} - [a(1-\overline{K})^2 - ca\overline{K}(1-\overline{K})]\overline{K},\end{aligned}$$

where we have used the fact that

$$\left.\frac{d\theta^*}{dk}\right|_{k=1} = \frac{d}{dk}\left.\left(1-\frac{\overline{K}}{k}\right)\right|_{k=1} = \overline{K}.$$

Starting from the corner $k = 1$, a reduction in k will increase social welfare if

$$\left(1+\frac{c}{2}\right)\overline{K} - [a(1-\overline{K})^2 - ca\overline{K}(1-\overline{K})]\overline{K} < 0,$$

which gives condition (12) and implies that the Lagrange multiplier λ satisfies (22). $\square$

Proof of Proposition 3. When the social cost of bank failure $c = 0$ it is immediate to check that the conditions $F(\theta^*, \lambda) = 0$ and $G(\theta^*, \lambda) = 0$ defined in (35) and (36) are satisfied for $\theta^* = \hat{\theta}$ and $\lambda = 1 + \hat{\delta}$. Hence comparing (6) and (7) with (19) and (20) we conclude that $k^*(\theta) = k(\theta, \hat{\delta})$ and $p^*(\theta) = p(\theta, \hat{\delta})$.

The analyze the effect of an increase in c we first compute

$$\frac{\partial G(\theta^*, \lambda)}{\partial c} = \int_{\theta^*}^{1} \frac{\partial k^*(\theta)}{\partial c}\, d\theta > 0,$$

and

$$\begin{aligned}\frac{\partial F(\theta^*, \lambda)}{\partial c} &= [2ap^*(\theta^*) + ca\theta^*]\frac{\partial p^*(\theta^*)}{\partial c} - (1 - p^*(\theta^*))a\theta^* - \lambda\frac{\partial k^*(\theta^*)}{\partial c} \\ &= \left[2\lambda\frac{a\theta^*(1+c)}{(2\lambda-1)}\right]\frac{\theta^*}{2(2\lambda-1)} - \lambda\frac{a(\theta^*)^2(1+c)}{(2\lambda-1)^2} - (1 - p^*(\theta^*))a\theta^* \\ &= -(1 - p^*(\theta^*))a\theta^* < 0.\end{aligned}$$

Hence an increase in c produces an upward shift the relationship between λ and θ^* implicit in both (35) and (36) (putting λ in the horizontal axis), which implies $d\theta^*/dc > 0$ (and an ambiguous effect on λ). Since for $c = 0$ we have $\theta^* = \hat{\theta}$, this implies $I^* = 1 - \theta^* < 1 - \hat{\theta} = \widehat{I}$ for $c > 0$, so aggregate investment will be lower under the optimal regulation.

Next using the condition that determines the equilibrium cost of capital in the absence of regulation (10) and the capital availability constraint (16) we have

$$\int_{\hat{\theta}}^{1} k(\theta, \hat{\delta})d\theta = \int_{\theta^*}^{1} k^*(\theta)d\theta = \overline{K}.$$

Using the result $\theta^* > \hat{\theta}$ we have

$$\int_{\hat{\theta}}^{\theta^*} k(\theta, \hat{\delta})\, d\theta + \int_{\theta^*}^{1} [k(\theta, \hat{\delta}) - k^*(\theta)]d\theta = 0,$$

which implies

$$\int_{\theta^*}^{1} [k(\theta, \hat{\delta}) - k^*(\theta)]d\theta < 0.$$

But by (6) and (19) we have

$$k(\theta, \hat{\delta}) - k^*(\theta) = \frac{a\theta^2}{2}\left[\frac{1}{(1+2\hat{\delta})^2} - \left(\frac{1+c}{2\lambda-1}\right)^2\right],$$

so it must be the case that $k(\theta, \hat{\delta}) < k^*(\theta)$ for all $\theta \in [\theta^*, 1]$, which proves that the optimal regulation requires banks to have more capital than they would in the absence of regulation. By (33) this in turn implies $p(\theta, \hat{\delta}) < p^*(\theta)$ for all $\theta \in [\theta^*, 1]$, so banks are safer than in the absence of regulation.

Finally, to prove that the optimal capital requirements will be binding we first show that $\lambda < 1 + \delta^*$. By the proof of Proposition 2, the first-order condition that characterizes the type θ^* of the marginal bank is

$$a(p^*(\theta^*))^2 - (1 - p^*(\theta^*))ca\theta^* - \lambda k^*(\theta^*) = 0.$$

This condition together the condition (23) that characterizes the equilibrium cost of bank capital δ^* under the optimal regulation gives

$$(1 + \delta^* - \lambda)k^*(\theta^*) = (1 - p^*(\theta^*))ca\theta^* > 0,$$

which implies $\lambda < 1 + \delta^*$. We want to show that the derivative with respect to k of the bank's objective function (34) evaluated at the optimal capital requirement $k^*(\theta)$ is negative, that is

$$2ap(\theta, k)\frac{\partial p(\theta, k)}{\partial k} < 1 + \delta^*.$$

But the first-order condition in Proposition 2 that characterizes the optimal capital requirements $k^*(\theta)$ is

$$2ap(\theta,k)\frac{\partial p(\theta,k)}{\partial k} + ca\theta\frac{\partial p(\theta,k)}{\partial k} - \lambda = 0.$$

Using the fact that $\partial p(\theta, k)/\partial k > 0$ by (33) and the result $\lambda < 1 + \delta^*$, this implies

$$2ap(\theta,k)\frac{\partial p(\theta,k)}{\partial k} = -ca\theta\frac{\partial p(\theta,k)}{\partial k} + \lambda < 1 + \delta^*,$$

as required. □

References

Allen, F., Gale, D., 2000. Comparing Financial Systems. MIT Press, Cambridge, MA.

Basel Committee on Banking Supervision, 2004. International Convergence of Capital Measurement and Capital Standards. A Revised Framework, Bank for International Settlements, Basel.

Basel Committee on Banking Supervision, 2010a. Basel III: A Global Regulatory Framework for More Resilient Banks and Banking Systems, Bank for International Settlements, Basel.

Basel Committee on Banking Supervision, 2010b. Guidance for National Authorities Operating the Countercyclical Capital Buffer, Bank for International Settlements, Basel.

Brunnermeier, M., Crockett, A., Goodhart, C., Persaud, A., Shin, H., 2009, Geneva Report on the World Economy 11: The Fundamental Principles of Financial Regulation, International Center for Monetary and Banking Studies, Geneva.

Daníelsson, J., Embrechts, P., Goodhart, C., Keating, C., Muennich, F., Renault, O., Shin, H., 2001. An academic response to Basel II. Financial Markets Group Special Paper 130, London School of Economics.

Gordy, M., Howells, B., 2006. Pro-cyclicality in Basel II: can we treat the disease without killing the patient? Journal of Financial Intermediation 15, 395–417.

Hanson, S., Kashyap, A., Stein, J., 2011. A macroprudential approach to financial regulation. Journal of Economic Perspectives 25, 3–28.

Holmström, B., Tirole, J., 1997. Financial intermediation, loanable funds, and the real sector. Quarterly Journal of Economics 112, 663–691.

Kashyap, A., Stein, J., 2004. Cyclical implications of Basel II capital standards. Federal Reserve Bank of Chicago, Economic Perspectives 1st Quarter, 18–31.

Repullo, R., 2005. Liquidity, risk-taking, and the lender of last resort. International Journal of Central Banking 1, 47–80.

Repullo, R., Saurina, J., Trucharte, C., 2010. Mitigating the procyclicality of Basel II. Economic Policy 64, 659–702.

Repullo, R., Saurina, J., 2012. The countercyclical capital buffer of Basel III: a critical assessment. In: Dewatripont, M., Freixas, X. (Eds.), The Crisis Aftermath: New Regulatory Paradigms, CEPR, London, pp. 45–67.

Repullo, R., Suarez, J., 2004. Loan pricing under Basel capital requirements. Journal of Financial Intermediation 13, 496–521.

Repullo, R., Suarez, J., 2013. The procyclical effects of bank capital regulation. Review of Financial Studies 26, 452–490.

Shleifer, A., Vishny, R., 2010. Unstable banking. Journal of Financial Economics 97, 306–318.

Stiglitz, J.E., Weiss, A., 1981. Credit rationing in markets with imperfect information. American Economic Review 71, 393–410.